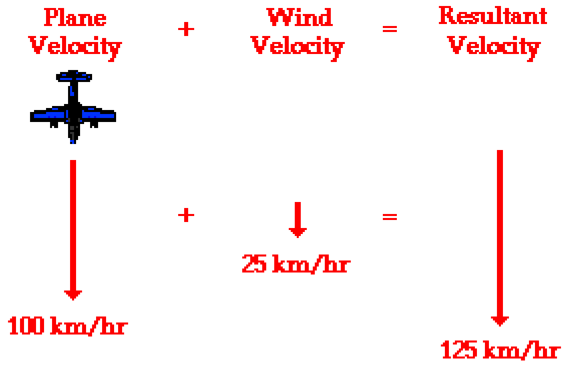
Relative Velocity and Riverboat Problems

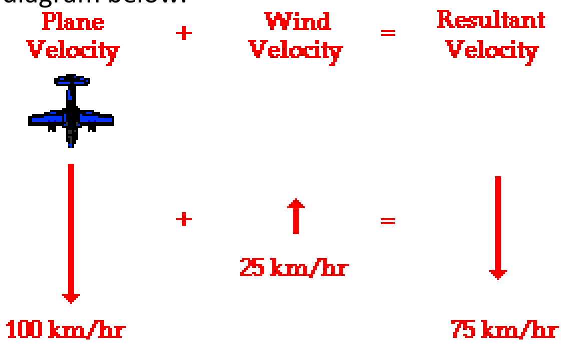
On occasion objects move within a medium which is moving with respect to an observer. For example, an airplane usually encounters a wind - air which is moving with respect to an observer on the ground below. As another example, a motor boat in a river is moving amidst a river current - water which is moving with respect to an observer on dry land. In such instances as this, the magnitude of the velocity of the moving object (whether it be a plane or a motor boat) with respect to the observer on land will not be the same as the speedometer reading of the vehicle. That is to say, the speedometer on the motor boat might read 20 mi/hr; yet the motor boat might be moving relative to the observer on shore at a speed of 25 mi/hr**. Motion is relative to the observer.** The observer on land, often named (or misnamed) the "stationary observer" would measure the speed to be different than that of the person on the boat. The observed speed of the boat must always be described relative to who the observer is.

To illustrate this principle, consider a plane flying amidst a tailwind. A tailwind is merely a wind which approaches the plane from behind, thus increasing its resulting velocity. If the plane is travelling at a velocity of 100 km/hr with respect to the air, and if the wind velocity is 25 km/hr, then what is the velocity of the plane relative to an observer on the ground below? The resultant velocity of the plane (that is, the result of the wind velocity contributing to the velocity due to the plane's motor) is the vector sum of the velocity of the plane and the velocity of the wind. This resultant velocity is quite easily determined if the wind approaches the plane directly from behind. As shown in the diagram below, the plane travels with a resulting velocity of 125 km/hr relative to the ground.



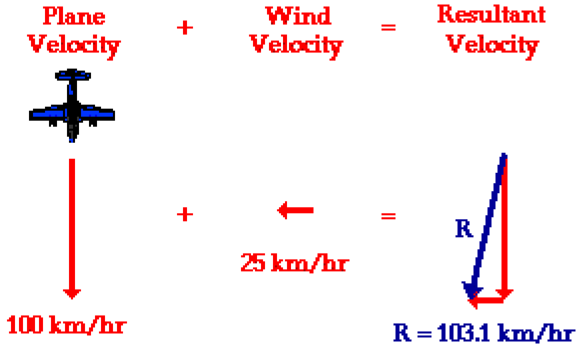
The plane travels with a velocity relative to the ground which is the vector sum of the plane's velocity (relative to the air) plus the wind velocity.

If the plane encounters a headwind, the resulting velocity will be less than 100 km/hr. Since a headwind is a wind which approaches the plane from the front, such a wind would decrease the plane's resulting velocity. Suppose a plane travelling with a velocity of 100 km/hr with respect to the air meets a headwind with a velocity of 25 km/hr. In this case, the resultant velocity would be 75 km/hr; this is the velocity of the plane relative to an observer on the ground. This is depicted in the diagram below.



Now consider a plane traveling with a velocity of 100 km/hr, South which encounters a side wind of 25 km/hr, West. Now what would the resulting velocity of the plane be? This question can be answered in the same manner as the previous questions. The resulting velocity of the plane is the

vector sum of the two individual velocities. To determine the resultant velocity, the plane velocity (relative to the air) must be added to the wind velocity. This is the same procedure which was used above for the headwind and the tailwind situations; only now, the resultant is not as easily computed. Since the two vectors to be added - the southward plane velocity and the westward wind velocity - are at right angles to each other, the Pythagorean theorem can be used. This is illustrated in the diagram below.



In this situation of a side wind, the southward vector can be added to the westward vector using the usual methods of vector addition. The magnitude of the resultant velocity is determined using Pythagorean theorem. The algebraic steps are as follows:

$(100\ \text{km/hr})^2 + (25\ \text{km/hr})^2 = R^2$

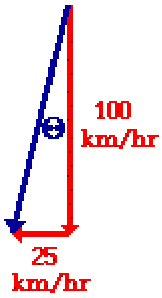
$10\,000\ \text{km}^2/\text{hr}^2 + 625\ \text{km}^2/\text{hr}^2 = R^2$

$10\,625\ \text{km}^2/\text{hr}^2 = R^2$

$\text{SQRT}(10\,625\ \text{km}^2/\text{hr}^2) = R$

$103.1\ \text{km/hr} = R$

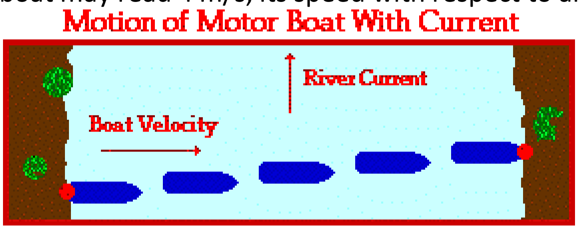
The direction of the resulting velocity can be determined using a trigonometric function. Since the plane velocity and the wind velocity form a right triangle when added together in head-to-tail fashion, the angle between the resultant vector and the southward vector can be determined using either the sine, cosine, or tangent functions. The tangent function can be used; this is shown below:



tan (theta) = (opposite/adjacent)
tan (theta) = (25/100)
theta = invtan (25/100)
theta = 14.0 degrees

Analysis of a Riverboat's Motion

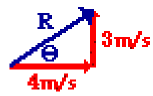
The affect of the wind upon the plane is similar to the affect of the river current upon the motor boat. If a motor boat were to head straight across a river (that is, if the boat were to point its bow straight towards the other side), it would not reach the shore directly across from its starting point. The river current influences the motion of the boat and carries it downstream. The motor boat may be moving with a velocity of 4 m/s directly across the river, yet the resultant velocity of the boat will be greater than 4 m/s and at an angle in the downstream direction. While the speedometer of the boat may read 4 m/s, its speed with respect to an observer on the shore will be greater than 4 m/s.



The resultant velocity of the motor boat can be determined in the same manner as was done for the plane. The resultant velocity of the boat is the vector sum of the boat velocity and the river velocity. Since the boat heads straight across the river and since the current is always directed straight downstream, the two vectors are at right angles to each other. Thus, the Pythagorean theorem can be used to determine the resultant velocity. Suppose that the river was moving with a velocity of 3 m/s, North and the motor boat was moving with a velocity of 4 m/s, East. What would be the resultant velocity of the motor boat (i.e., the velocity relative to an observer on the shore)? The magnitude of the resultant can be found as follows:

$(4.0 \text{ m/s})^2 + (3.0 \text{ m/s})^2 = R^2$
$16 \text{ m}^2/\text{s}^2 + 9 \text{ m}^2/\text{s}^2 = R^2$
$25 \text{ m}^2/\text{s}^2 = R^2$
$\text{SQRT} (25 \text{ m}^2/\text{s}^2) = R$
$5.0 \text{ m/s} = R$

The direction of the resultant is the counterclockwise angle of rotation which the resultant vector makes with due East. This angle can be determined using a trigonometric function as shown below.



tan (theta) = (opposite/adjacent)
tan (theta) = (3/4)
theta = invtan (3/4)
theta = 36.9 degrees

Given a boat velocity of 4 m/s, East and a river velocity of 3 m/s, North, the resultant velocity of the boat will be 5 m/s at 36.9 degrees.